TEACHING THE ESSENTIALS OF ARITHMETIC

by

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TO

THE YOUNG ARITHMETICIAN

WHOM

I KNOW BEST AND LOVE BEST—

MY DAUGHTER

BRONWEN

A few strong instincts and a few plain rules.

WORDSWORTH.

Pursued in the spirit of a philosopher and not of a shop keeper, arithmetic has a very great and elevating effect, compelling the soul to reason about abstract number, and rebelling against the introduction of visible or tangible objects into the argument.

PLATO: *The Republic.*

And what of all this? Marry, read the book and you shall know; but read nothing except you read all. And why so? Because the beginning shews not the middle, and the middle shews not the latter end.

THOMAS DELONEY: *The Gentle Craft* (1543-1600).

PREFACE

Rarely is a teacher satisfied with the arithmetic of his class. He is not allowed to be. Somebody always has something to say about it—generally something unpleasant. If it is not a colleague, it is probably an inspector; if it is neither of these, it is a parent or an employer. And even when the criticism is not aimed at a particular person, but lies couched in a general report on a public examination, or in a survey of a prescribed area, or in an utterance from the platform or the press, its generality makes it none the less a disturbing force. The circle of disturbance is larger, that is all. Like slugs fired from a blunderbuss, the discharge hits many though it kills none. Not that one can object to these general criticisms. On the whole they do good. What the teacher has to do is to grow a skin thin enough to let him know when he is hit, but thick enough to protect him from serious wounds. It is true that many of these blunderbuss criticisms scatter through the empty air and hit nothing (this is specially true of those that come from the employer and the press), but some of them reach home. Some of them touch real points of weakness. And these, as a rule, are the charges that are common to all the indictments.

What the critics say, and say with one voice, is that the fundamentals are weak; that our children are badly grounded in the very ABC of arithmetic. They don't complain that the pupils can't work compound proportion; but they do complain that they can't work compound multiplication. Nobody grumbles at their not knowing logarithms; but everybody grumbles at their not knowing the multiplication table, and not being able to add and subtract with that ease and accuracy which ordinary life demands.

There is no lack of excellent textbooks on the market. There are Common-sense Arithmetics which are chock full of common sense, and Efficiency Arithmetics which cannot fail to make a lad efficient if he can only be induced to work the examples. And there are many others equally good, and, to all appearances, equally worthy of their titles. Nor is there a lack of good books on the teaching of arithmetic. It is difficult to conceive a clearer or more comprehensive book than Mr. F. F. Potter's, or a more helpful and charming little book than Miss Jeannie B. Thomson's. And yet our pupils bungle at the very rudiments of the subject. The only conclusion I can come to is that the textbooks are too good for the pupils. The arithmetical fare we offer them is ill-suited to their digestions: it is either too rich or too much. What they need is a simpler diet, a diet which they can manifestly turn into sound bone and muscle, a diet for which a keen appetite is the best sauce. From what I can discover, the appetite at present is none too good.

As for our books on method, they all suffer from an excess of logic and a dearth of psychology. They follow tradition in stressing the rational side of arithmetic. After making a few concessions to the immature mind, which may be allowed to gain its first notions of number from bricks or beans, they hasten to apply general principles to particular instances. They proceed deductively. Every step at every stage has to be reasoned out. No unexplained process is allowed to be taken on trust and used on the sole ground that the process works. No lumps of knowledge, however useful they may be as they are, are allowed, even for a while, to escape the grinding of the logical machine. And arithmetic is almost made to appear as a mere branch of deductive logic.

This is the English view. But there is another view which finds its clearest utterance in Thorndike's book on *The Psychology of Arithmetic.* This book sets the tune to which all American educators now dance. Arithmetic is presented as an inductive science. Reasoning starts with the concrete fact and ends with the concrete fact. Children learn arithmetic by working sums. The justification for the mode of procedure is that the answer is right. The ground for believing the answer to be right is the word of the teacher, or the result got by reversing the process, or, in the last resort, the irrefutable evidence afforded by counting. The child does a thing first and understands it after. Doing is the important thing; and practice in doing—the practice that, "line upon line, here a little and there a little," fixes deeper and deeper a series of habits. Arithmetic is in fact not so much an application of broad general principles as as organisation of habits.

Thus we have on the one hand the English view that arithmetic is logic, and the American view that it is habit. The contrast is interesting and significant; but it is not new. It resembles, in fact, the antithesis between the Platonic and the Aristotelian views of virtue. To Plato virtue is knowledge; to Aristotle it is habit. To Plato it is an intellectual grasp of the consequences of our acts; to Aristotle it is the practice of choosing the mean between two extremes.

These differences in emphasis and in outlook are not of mere theoretical import: they vitally affect practice. They prescribe what we shall teach, how we shall teach it, and how we shall test it. Hence the study of these two contrasting attitudes may yield us the key to the solution of our difficulties. I think the key is really there; but it is a duplicate key, or rather a multiplicate key. I had myself on certain vital points come to pretty much the same conclusions as Thorndike long before I knew what Thorndike's conclusions were. So no doubt had many of my readers. The outstanding fact is that the habit element in arithmetic has in recent years in England been obscured by ill-founded views on the place and function of intelligence in the study of the subject. And one of the main aims of this little book is to remove the obscurity, and to reveal the role of habit in the mastery of arithmetic; to show that in the erection of a sound fabric of knowledge, though intelligence may be the architect and builder, yet it is habit that gives cohesion to the bricks and adhesion to the mortar, and to attempt to build without its aid is worse than trying to build *on* sand: it is trying to build *with* sand.

PREFACE

Although this book aims at reconciliation, it must not be thought that I am mainly concerned in balancing pros and cons, or in expanding the sentence: "Much may be said on both sides." I am not. I don't sit on the fence: I come down definitely on one side or the other. Let me cite a few instances. I think the method of teaching subtraction by decomposition a vicious method. I am convinced that the policy of shirking long division till late in the course and substituting division by factors is wasteful and ineffective. I hold that compound multiplication by factors is a clumsier method than direct multiplication in one line, and that the unitary method of working proportion is more cumbersome that the fractional method. And I am a great believer in the King's highway—in having one good standard method of working a given type of sum. I have observed that those who show too eager a desire to avoid the beaten track and discover short cuts often come to grief. They either lose their way or arrive late. Meanwhile their more pedestrian classmates who have gone the longest way round have really found the nearest way home.

We must distinguish (if we can) between new ideas that have come to stay and new ideas that arise from chance and change—ideas which are in the fashion, and have in consequence a certain air of smartness, but come off badly when subjected to the wear and tear of the classroom. The worst of it is, when one idea gets into fashion it pushes another idea out of fashion. And the other is often the better of the two. It was the fate that befell "equal additions" when "decomposition" got into vogue; it is a disaster that threatens to befall that good old method of multiplying decimals—counting the decimal places., The ground of the threatened taboo seems to be that if this old method is continued, children will be getting their sums right all over the place, and there will be no need for them to acquire some particular piece of pedantic ritual. There are other rules, too, which have recently come under the ban of the doctrinaire, rules such as proportion, practice, alligation, and the reduction of problems to types. Such bans are, as a rule, quite unreasonable: they have no basis in theory, no justification in practice.

This bold (and bald) confession of faith may seem to contain a touch of defiance: I may seem to be trailing my coat. In reality I am merely trying to be honest, trying to tell the reader what he may expect to find in the pages that follow. The one question which I have steadily kept before me is: What are the methods which, while theoretically sound, succeed best with children? What, in other words, are the methods which enable the young learner to get the most sums right in the shortest time? The evidence I seek is that of experiment and experience, and my court of appeal is the classroom.

There is no branch of study which is so dominated by examinations as arithmetic. For it is the most examinable subject in the curriculum; and being the most examinable it is the most examined. Whatever the type of general examination, arithmetic is sure to be brought in. Even the intelligence examination is not exempt. Indeed, it has introduced a new model—a Parisian model, popularised if not invented by Alfred

Binet. The technique of the mental test demands that a question should be brief, simple, and unequivocably scorable. It should, if possible, carry one mark or none. This technique has already begun to influence the examination question in arithmetic: it has reduced its size and its complexity. And in so doing it has led to the invasion of new territory. Let us look for a moment at the old order of things; which is still, in the main, the prevailing order of things. The traditional test in arithmetic consists of a number of questions, each of which takes from ten to fifteen minutes to answer. The examination sum is a ten-minutes sum. If another paper is set, it consists of mental arithmetic questions with about ten seconds allowed for each. The range between the ten-seconds sum and the ten-minutes sum is entirely untouched. Here is a huge tract of No-man's-land the existence of which is ignored by the examiner. And yet in everyday life this is the very region in which most of our calculations lie. If it be objected that the neglect applies to examination sums only, it may be replied that classroom sums are echoes of examination sums; for nearly all the arithmetic done in our schools (I state it as a fact, not as a fault) is done in preparation for some sort of examination. A change in the examination must mean a change in the teaching, and a book which deals with teaching must take cognisance of the fact.

I do not propose to discuss why we teach arithmetic, because that is a question which nobody really asks with a genuine desire to hear the answer. When he asks it at all, it is with a desire to tell the answer, not to hear it. The teacher never asks it. He knows he has to teach the subject in any case, and he is content to leave it at that. But there are other questions which he is constantly asking. How, for instance, may John Smith, a decent enough lad in his way, but one who seems to regard arithmetic as something "from which the mind instinctively retires"—how may John Smith be brought to take an interest in the subject? The question of motivation in fine is not fictitious: it is real, vital, and of perennial urgency. Pedagogical textbooks ignore it; they assume that John Smith does not exist; they teach in effect that little children take to arithmetic as ducks take to water. All we have to do is to deal out to the little dears a nice set of sums, and they will work them with avidity, and even ask for more. And their minds are all agog for explanations, eager to know the why and the wherefore of all the processes they employ. No teacher holds these views. Many of them indeed are filled with surprise when they find a child who will cipher, not as a task, but as a joy, and they will secretly point him out as a prodigy—as something strange and unnatural. And yet the subject is full of romance, as Professor Spearman has recently been pointing out. "How comes it," he asks, "that this mathematics, controller of destinies, source of delight, fount of emotion, breeder of romance, has arrived at being almost universally besmirched with the attribute of 'dull'?" He goes on to say: "For my part, I would mainly attribute this huge miscarriage of justice to the manner in which the subject is being taught in school." (*The Outlook*, December 24, 1927). That Professor Spearman is right I have little doubt. But whether he is right or wrong, he raises a question which clamours for an answer.

As I have already said, I have attempted a reconciliation—or, if you Jike, a compromise—between the extreme English view and the extreme American view. There is another *entente* which I have at heart: a more complete understanding between the primary school, and the secondary school, and a better co-ordination of methods. The methods are sometimes sadly at variance. In the primary school, for instance, the pupil is taught to begin to multiply with the units figure of the multiplier; in the secondary school he is taught to begin at the other end. This seems a small matter, but it is big with consequence to the higher half of the arithmetic course. And in this instance the experimental evidence is on the side of the secondary school. There is a distinct gain in beginning to multiply from the weightier side.

At other times it is the primary school that is right and the secondary school wrong—right and wrong being determined as before by experimental evidence. Primary-school children are taught a simple and extremely efficient method of multiplying decimals—the method of adding decimal places. It is a sound, rational method, easy to teach, easy to understand, easy to apply. It has all the desirable characteristics of a standard rule-of-thumb method. It depends, too, upon a principle which is of wide application in the higher branches of arithmetic. And yet, for some reason or other, this sane and simple method is in many secondary schools regarded as arbitrary and irrational, and is as rigidly suppressed as though it were some deadly heresy destructive of all mathematical integrity of mind. And in its place is put a cumbersome method of multiplying known as the "standard form" methbd—a method which takes nearly twice as long and is about half as accurate as the old-fashioned method. At first I thought there was in "standard form" some mysterious virtue which had escaped my notice. But after discussing the matter with the best mathematicians I know, I find there isn't. I was assured by one master that it was a method invented to prevent parents from helping children with their homework. The real purpose, however, was to secure uniformity of method among entrants to secondary and public schools. The inventors of the method should try again, and invent a better one.

The obligations I have to acknowledge are many. The writers to whom I am chiefly indebted are Augustus De Morgan (the delight of the connoisseur), Sir Oliver Lodge, and Professors Spearman, Nunn, and Thorndike. The main stimulus, to the writing of this book, however, has been my own experience in schools. Such views as I put forward are the harvest of many years of teaching, testing, observing, and experimenting. The fact that I have more respect for experiment than for opinion—including, I hope, my own opinion—does not prevent me from being inordinately pleased when I find myself in agreement with my friends; especially when those friends happen to be both mathematicians and philosophers. I am happy therefore to record that Professor Spearman has read in manuscript the chapters on mathematical reasoning, and Professor Nunn the chapter on incommensurables, and that both concur generally in the views therein expressed.

I am deeply indebted to my colleague Mr. E. P.

Bennett, who has read through the manuscript with much care, and has given me the benefit of his knowledge and experience. He has also been kind enough to correct the proofs. It is impossible to express adequately my appreciation of the ready and generous help I have received in my arithmetic investigations for this and other books from teachers of all grades. Chief among them are Mr. A. Wisdom, Mr. J. G. Robson, Mr. T. H. Elliott, Mr. H. R. Neilson, and Mr. H. H. Spratt. Finally, I owe an accumulated debt of gratitude to Mr. W. Stanley Murrell, the Manager of the University of London Press, who has wisely counselled me for many years, and has with great skill piloted a number of my books through the straits of printing and binding out into the open sea.

P. B. Ballard.

Chiswick.

April 1928.

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CHAPTER I

INTELLIGENCE AND HABIT

What a piece of work is man! How noble in reason! how infinite in faculties!

SHAKESPEARE: *Hamlet.*

A house built upon reason is a house built upon sand.
Knowledge must become automatic before we are safe with it.

SAMUEL BUTLER: *Life and Habit*.

Whenever thought is necessary, it is to be exercised vigorously, but it should not be wasted over simple mechanical operations.

SIR OLIVER LODGE: *Easy Mathematics*.

A GENERATION ago many of the most vocal members of our profession made much ado about Intelligence. It was their great word. They used it in all their arguments; they stressed it in all their speeches. The cultivation of intelligence was the grand aim and purpose of education, and the value of each branch of study was to be measured by the extent to which it furthered this great purpose. And a method of teaching was good or bad according as it hastened or hindered the growth of intelligence. Intelligence had thus become a touchstone as well as a watchword. It is true that the meaning of the word was a little obscure, and the sense in which it was used was prone to vacillate. Sometimes it meant this, and sometimes it meant that. In a general way, however, it included the higher, the more rational, the more distinctively human operations of the mind, and excluded those powers which we share with the beasts that perish. And as intelligence was the noblest function of the human mind, so was memory, in all its manifestations, the most ignoble. Doubly ignoble was it when it took the form of habit memory—memory that had become so deeply embedded in brain and nerve as to be organic, to be part and parcel of the human organism. For by this time the process had become so mechanical that it almost worked of its own accord. Man made in God's own image had been reduced to the level of a machine. And so, while intelligence was glorified, automatism was vilified; while one was lauded to the skies, the other became an object of derision and scorn.

It was an inspiring doctrine, well calculated to capture the young and generous mind. The very catchwords and slogans were full of appeal: "Capacity, not content"; "Power which brings knowledge, not knowledge which may or may not bring power"; "An agile mind rather than a full mind"; "The only habit which the child should be allowed to form is the habit of contracting no habit at all" (this from Rousseau's *Émile*). These are attractive sentiments. The only objection to them is that they won't work. They are out of touch with reality. They are based on a false notion of human nature, and of the rôle of intelligence on the one hand and of habit on the other in the building up of human knowledge. The result is that they either do not influence practical teaching at all, or influence it to its hurt.

The doctrine involves the belief that brain-power can be generated by special studies—that mental gymnastics of a particular kind can produce a mental gymnast of a general kind. The technical term for this theory is "formal training." The first man to shake English teachers out of their complacent trust in this theory was Sir John Adams, then plain John Adams of Glasgow. In his little book on *Herbartian Psychology* published in 1897 he subtly and humorously insinuated such doubts into the minds of his readers that they soon ceased to talk about cultivating the faculty of intelligence. Others followed his lead. They took his matter though they changed his manner. Dr. Hayward preached the new Herbartian creed from the platform and from the professional press with all the ardour of an old Hebrew prophet. As Jonah predicted the downfall of Nineveh, so did Hayward proclaim the downfall of Formal Training, and many of his hearers thought he was talking nonsense; they now know he was talking plain common sense—with perhaps a touch of exaggeration to make it picturesque. Meanwhile Mr. Winch, with a keen eye for the essentials of a problem, had been putting the matter to the test of experiment. He re-christened the problem and called it the problem of transfer. The question at issue appeared in this form: Is ability acquired in one function transferred to another function, distinct and dissimilar? A function gains with practice. Does it share the gain with others, or does it keep it all for itself? Mr. Winch's conclusions were that though it keeps the huge bulk for itself, it does probably share a little of it with others. If there is any transfer, it is very small.

In the meantime, researches of a different kind were being pursued by Alfred Binet in France and by Professor Spearman and Dr. Cyril Burt in England. These pioneers were soon joined by other workers in other lands, all intent on one task, the task of measuring that mysterious something which went under the name of intelligence. Absurd as seemed the attempt to measure so elusive a thing, the researchers hoped that the very attempt to measure it would enable them to identify it, to pin it down, to define its nature and its limits. And their hopes have largely been realised. They have been able to show that intelligence manifests itself most unmistakably in those processes which are commonly called reasoning; that it is scarcely influenced at all by environment and training; and that it mainly determines a child's capacity to profit by the lessons he receives at school. Incidentally it has been brought to light that this "intelligence" of the psychologist is not quite what the man in the street means by intelligence.

What then becomes of the theory that the main object of teaching arithmetic, or indeed teaching anything else, is the cultivation of intelligence? When a teacher claims that he is achieving this object he does not quite mean what he says. What he really means is that he makes his pupils use what intelligence they already have—makes the child with five talents use his five, and the child with one talent use his one. That is the most he can do. He cannot by any species of jugglery turn a one-talent child into a two-talent child.

Intelligence is the mind's original capital (as Dr. William Garnett once phrased it), and the vital question is: Does it accumulate simple interest or compound interest? The out-and-out opponents of formal training—whole-hoggers like Dr. Sleight—cling to the simple-interest theory. More cautious investigators incline to the compound-interest theory—with this important reservation: the amount of interest that is added to the original capital is very small in comparison with the capital itself. The whole question is complicated by the fact that intelligence is not the whole of the mind's native endowment. A man is born with certain specific abilities as well as general ability; and these specific abilities are eminently trainable. I am here, however, dealing with general ability only, and am trying to show that the teacher's main concern with the intelligence of his children is to ensure that none of the mind's capital is unused, that none of the talents is wrapped up in a napkin.

As the Victorian theorists' views on the cultivability of intelligence were wrong, so also were their views on the worthlessness of habit. It would not be difficult to turn the tables on the last generation and sing the praises of habit at the expense of intelligence. Indeed, one of their own contemporaries has already done so. Samuel Butler's *Life and Habit,* which appeared in 1878, may be regarded as a long and brilliant essay in praise of automatism. Automatism is the final flower of knowledge—the goal to which all knowledge drives. And knowledge becomes more perfect as it becomes less conscious. The conscious knower is the bungler; the unconscious knower is the expert.

Let us consider what happens when a man learns to do something—to ride a bicycle, for instance. His first attempt is wild and fervid. He begins full of hope, even if he ends full of bruises. His mind is at high tension, his thinking at white heat. And the more he blunders the more furiously he thinks. But in the course of time his failures get fewer and fewer and his success more and more assured. A complex habit is gradually being formed; and as the habit gets stronger, the conscious control gets weaker. As automatism gets driven in, intelligence gets driven out. When the bicycle has been completely mastered it can be ridden with the minimum of attention and the minimum of volition. The whole process has been sinking more and more into the unconscious. That is what the Frenchman had in mind when he described education as the turning of the conscious into the unconscious.

Habit formation is therefore a beneficent thing. Far from being a deadening and enslaving process, it is a process of emancipation. All along the line there is a gradual releasing of energy, which is thus set free to function in new directions. And not only is it liberated, but it is provided, in the habit itself, with new material in which to work. Habit is thus seen to be a means of mental economy. It funds our knowledge and gives it security. So far as effective action is concerned, we are much safer in the hands of habit than in the hands of intelligence. Nature herself has never dared to entrust the vital physiological functions to the control of the intellect. The heart beats and the blood circulates with an automatism which is absolute and complete.

One characteristic of an automatic act is that it goes best by itself. If we poke thoughts into it, it goes wrong. We walk best when we are not thinking about walking; when we think out the order of the various movements by which we dress of a morning, we arrive late at breakfast.

CHAPTER II

EDUCTION

He thrids the labyrinth of the mind.

TENNYSON: *In Memoriam*.

THE reader with a turn for philosophy may justly complain that in the previous pages I have afforded him many glimpses of the familiar and not a few of the obvious. I will now, however, shift the argument to newer ground. As I have already stated, the main business of intelligence is reasoning. But what is reason? The mere fact that we still argue over the question, Can animals reason? indicates the general fogginess of our opinions on the matter. It isn't that we don't know what animals can do, but that we don't know whether to call it reasoning. The one thing upon which all are agreed is that in reasoning we somehow or other derive a new idea from old ideas—and this without further recourse to experience. Whether consciously or unconsciously, whether spontaneously or by deliberate effort, we "draw out" from something that is given something which is not explicitly but only implicitly given. The old term for this "drawing out" is "inference" or "deduction." Professor Spearman has given it the more appropriate term "eduction," and has defined it more closely. And it is this eductive logic of Professor Spearman's that I wish to apply to the reasoning processes in arithmetic. I select it on the simple ground that it is the only sort of logic that fits.

The logic that held the field for over a millennium was the deductive logic of Aristotle. Its essential form is the syllogism, of which the following is simple example:

All insects have six legs;
All bees are insects;
Therefore all bees have six legs.

It was thought that all true reasoning conformed to this type; yet every attempt to press mathematical reasoning into the syllogistic mould has signally failed.

In modern times it has been realised that deductive logic is only part of the process by which we arrive at truth. The larger process is called induction. It is the method of scientific inquiry. It begins earlier than deduction and includes it. It begins, not with the statement that all insects have six legs, but with the evidence for that statement. It begins, in fact, with particulars and not with generalities. The pertinent fact, however, is that it has failed, just as the earlier logic has failed, to explain the essential nature of mathematical reasoning.

The failure of traditional logic is probably due to the fact that it did not carry its analysis far enough: it had not arrived at the elemental units. It had reached the molecules, but not the atoms; or, to be ultra-modern, the atoms but not the electrons. The honour of finding the electrons belongs to Professor Spearman. His electrons are "fundaments" and "relations."

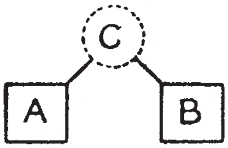

FIGURE 1

Let A in Figure 1 (these diagrams are Dr. Spearman's own) represent the number 2, and B the number 8. Having got a mental grip of A and B, we can cognise or "educe" a relation *(c)* between them. C may be "smaller than," or "¼ of," or "the cube root of," or "6 less than." The actual relation educed will depend on the task the thinker has in hand: he will select the one pertinent to his purpose. If, again, A is 5 + 2 and B is 7, then C is "equal to." This relationship of equality plays an important part in mathematics, The multiplication table is a convenient list of the more useful fundaments bearing this relationship. So are the other tables. In the solution of equations and the simplification of fractions the relationship of equality with its well-known symbol (=) dominates the whole proceedings.

In addition to the first kind of eduction, the eduction of relations, there is a second kind, the eduction of correlates. In Figure 2, A and C being given, we have to educe B, the missing fundament. If, for instance, A is 5 and C is "half of," we know that B must be 10. This is the eduction of a correlate, for the educed fundament

B is a correlate of the given fundament A. Each item of the multiplication table represents an operation which comes under the second principle. In the statement "4 times 7 are 28," 7 is the given fundament, "taken 4 times" is the given relation, and 28 is the educed correlate.

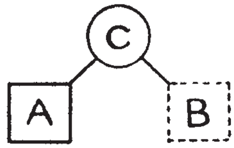

FIGURE 2

It must not be thought that the material on which the two eductive processes work is as simple as the above exposition would lead us to believe; for the product of one eduction may become a fundament of another. If A, B, C, D in Figure 3 are the fundaments from which we start (we must start somewhere), the relations between them may become the basis of new eductions. The figure indicates but a few of the possible relationships; and merely suggests their possible complexity. We may indeed pile up an indefinite number of relations

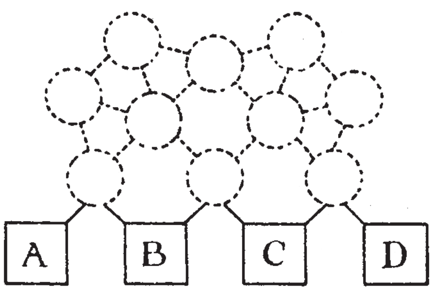

FIGURE 3

and correlates and form a huge fabric of constructive thought which terminates in notions widely removed from the elementary notions from which we started. Nor must it be thought that the construction of this fabric applies to reasoning only. It operates over the whole field of cognition, from the following of a cinema film to the understanding of a picture by Velasquez, a symphony by Beethoven, or a play by Shakespeare.

These two eductive processes pervade the whole of our thinking: they are the two steps by which the mind marches into new realms of thought. Direct experience and memory provide the solid starting-ground, but the forward thrust, the life and movement of the human spirit, come from eduction. As the verb is the soul of the sentence, so is eduction the soul of reason; and as the verb cannot live without the noun, so does eduction owe its very existence to the tributary service of sense and habit. And not habit only—the fixed and facile stage of memory—but every form of memory, every form of reproduction. Our ordinary thinking is a kaleidoscopic medley of eduction and reproduction. It is reproduction that gives the grist; it is eduction that does the grinding. And without grist there can be no grinding. Why is it that we are able to rear in the mind the sort of edifice pictured in Figure 3? Eduction is a narrow process; it cannot occupy a broad stage: it needs the spot-light. And the only way it can build high is by accepting the services—the menial services if you like—of memory. Memory hands over the products of previous eductions and enables the actual eductive activity to work among the higher storeys. If it is kept at work in the basement, the higher storeys will never be reared.

The moral of all which is: Let children learn the multiplication table so that it can be reproduced, item by item, with mechanical precision and promptitude; fix the routine of the simple rules so that they absorb the minimum of creative thought; foster the formation of useful habits so that intelligence may be kept at work in its proper sphere. Habit is a servant; see that it is a good servant. Intelligence is a master; see that it is not allowed to concern itself too much with life below-stairs.