LESSONS IN EXPERIMENTAL AND PRACTICAL GEOMETRY

by

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PREFACE

TO give to a young pupil clear mental pictures should be the first object of geometrical teaching: to enable him to express geometrical ideas in the form and order required by strict deductive reasoning is a second and distinct object. Experience shows that these two aims may to some extent be separated with advantage; and accordingly Formal Geometry is now very generally preceded by a short preliminary course of practical and experimental work.

In the preface to our *School Geometry* it is suggested that a suitable introduction to that book would consist of "Easy Exercises in Drawing to illustrate Definitions; Measurements of Lines and Angles; The Use of Compasses and Protractor; Problems on Bisection, Parallels, Perpendiculars; The Use of Set Squares and the Construction of Triangles and Quadrilaterals: these problems to be informally explained, and the results verified by measurement. Concurrently there should be Exercises in Drawing and Measurement designed to lead inductively to the more important Theorems of Part I." It is the purpose of these *Lessons* to supply such an introductory course.

To this scheme we have added very simple chapters on Areas, on Circles and Polygons, and on the Forms of some Solid Figures; but it is not intended that these Sections should necessarily be taken before demonstrative geometry is begun.

This experimental and constructive work should not be allowed to keep a pupil back. He may probably be put to it six months or a year before a start can profitably be made with geometry of a more formal kind; and when the latter stage is reached, his practical knowledge should not only add life and interest to his theoretical work, but greatly accelerate its progress.

In each Section more exercises are provided than Teachers are likely to need for a first course: the rest may be taken afterwards with the corresponding propositions in the *School Geometry,* to which this little book is intended as a supplement as well as an introduction.

H. S. HALL

F. H. STEVENS

***December* 1904**

CONTENTS

NECESSARY INSTRUMENTS

The pupil should be provided with the following instruments and apparatus:

1. A flat ruler, one edge being graduated in centimetres and millimetres, and the other in inches and tenths.
2. Two set squares; one with angles of 45°, and the other with angles of 60° and 30°.
3. A pair of pencil compasses.
4. A pair of dividers, preferably with screw adjustment.
5. A semi-circular protractor.

The instruments referred to above in Nos. 1 to 5 are supplied in Macmillan's Sets of Mathematical Instruments. The Elementary Set, on card, 3d. net. In Metal Pocket Case: The School Set, ls. net; The Beginner's Set, 1s. 6d. net; The Junior Set, 2s. net; The Senior Set, 2s. 6d. net.

6. Tracing paper. Squared paper.

It is also very desirable that pupils should have an opportunity of seeing and handling Models of the simpler Solid Figures.

A set of Models for use with this book has been specially prepared, and may be obtained (price 6s., in box, including carriage to any part of the United Kingdom) direct from the manufacturer

G. Cussons
The Technical Works
Lower Broughton
Manchester

CHAPTER I

SOLIDS, SURFACES, LINES

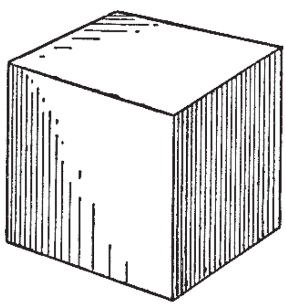


Fig. 1.

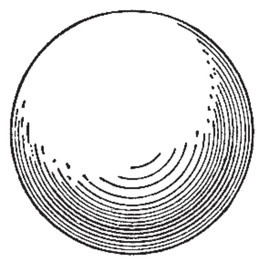


Fig. 2.

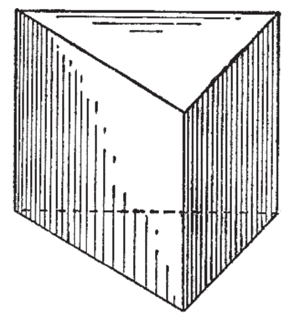


Fig. 3.

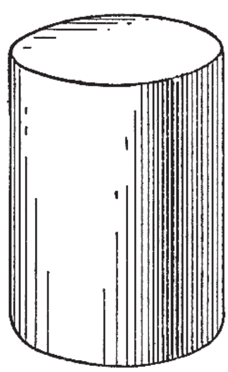


Fig. 4.

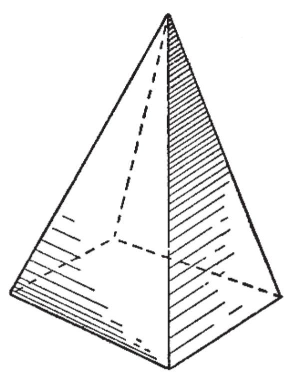


Fig. 5.

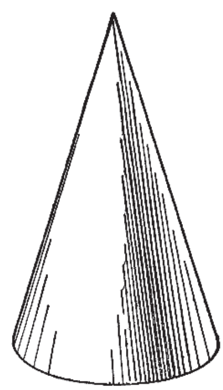


Fig. 6.

We have here some wooden models of what are called **solids** or **solid figures,** and they are differently named according to their shapes. That, for instance, of which a drawing is given in Figure 1, is called a **cube;** that shown in Figure 2 is a **sphere;** that in Figure 4 is a **cylinder;** and that in Figure 5 is a **pyramid.**

The *outside* of these solid models, the part which we see and touch, is called the **surface**.

Sometimes the surface of a solid is all in one piece, as in the sphere (Figure 2). Sometimes it consists of several parts: for instance in the cube (Figure 1) the surface consists of six parts, all flat; these are called **faces.** Again, in the cylinder (Figure 4) the surface consists of *three* parts, one rounded and the other two flat. Once more, the surface of the cone (Figure 6) is in *two* parts, one rounded and running to a point, the other flat.

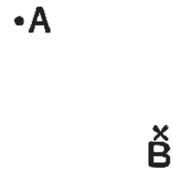
Let us now see how two neighbouring parts of a surface meet. They meet in **edges** or **lines;** and these lines are sometimes *straight,* and sometimes *curved.* In the prism and pyramid (Figures 3 and 5) two neighbouring flat faces meet in a *straight* line; while in the cylinder (Figure 4) the rounded part of the surface meets each flat end in a *curved* line.

How do the *edges* of a solid meet? If two edges meet at all, they meet at a **point***;* as you will see if you look at the edges of a cube or pyramid (Figures 1 and 5).

You now know what a solid is, and what a surface is; and you have learned that surfaces, or parts of a surface, meet in lines and that lines meet in points. We have now to see how lines and points are represented in geometry; how *straight* lines are distinguished from *curved* lines; and how flat surfaces are distinguished from rounded ones.

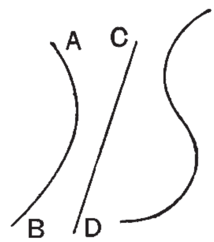
Points. The smallest dot you can make on your paper with a sharp pencil, or with a fine needle, will give you an idea of what is meant by a geometrical point. A point is so minute that we do not think of its length, breadth, size, or shape: all we have to consider is its *position.*

As we have seen, a point marks the place where two lines cross one another. Points are named and distinguished from one another by attaching letters to them: thus we speak of the point **A**, or the point **B**.



Lines. We represent a line by drawing the point of a sharp pencil over a surface, such as a sheet of paper: this shows that *a line is traced out by a moving point.*

Several kinds of line are shown in the margin.

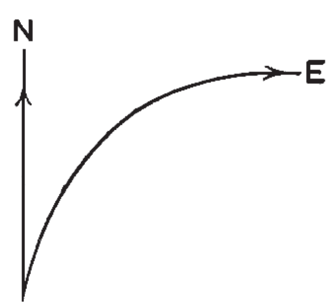


All lines have *length,* some more, some less; but the *breadth* of a well drawn line is so small that no notice is taken of it in geometrical work: indeed, the finer your pencil-trace, the better it represents a line.

What we have to consider in a line is its *length* and *position,* and whether it is *straight* or *curved.* A line is named by two letters: thus we speak of the line **AB**, or the line **CD**.

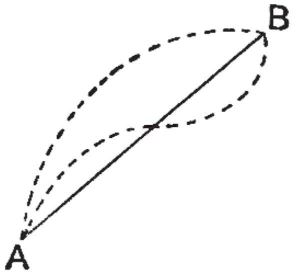
Straight lines. No doubt you already know the meaning of the word *straight* well enough to give examples of straight lines. A very fine thread tightly stretched is a good instance of a straight line; so are the edges of the set squares which you are to use as rulers. But *straightness* needs some further illustration.

(i) When you walk along a winding lane you notice that your direction is continually changing; and if, for instance, you faced North when you started, you may presently find yourself facing East. But when you walk along a *straight* road, there is no change of direction as you

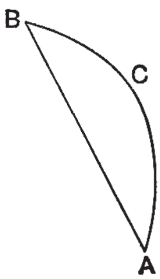


advance; and if you faced North at starting, you will continue to face North.

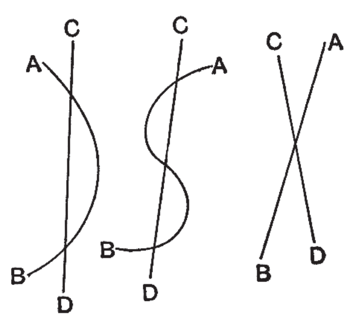
(ii) In a field there are two trees whose positions are marked by the letters **A** and **B**. Suppose you wish to go from one tree to the other by the *shortest* way. You can see at once what course you must steer. You must go *straight* from **A** to **B**. There are numberless *curved* lines along which you could go from one tree to the other, but the shortest way of all is the *straight* line. You notice that we have said *the* straight line; for you can see for yourself that there can only be *one straight* line leading from **A** to **B**.



(iii) A strip of ground has been enclosed by two fences. One of these, **AB**, is straight: can the other be straight also? Clearly not; for we have already seen that there cannot be more than one *straight* line between **A** and **B**, though many curved lines such as **ACB**.



(iv) We will draw a curved line, and call it **AB**; then we will rule a straight line **CD** across it. You see that you can place your ruler so that the straight line will cut the curved one at *two* points, perhaps even more than two. Now take a *straight* line **AB**, and rule another straight line **CD** across it. Can you now place your ruler so as to cut **AB** in more than one point? You will soon find that you cannot.



Let us now put together what we have learned about straight lines.

(i) *A straight line has the same direction throughout its length.*

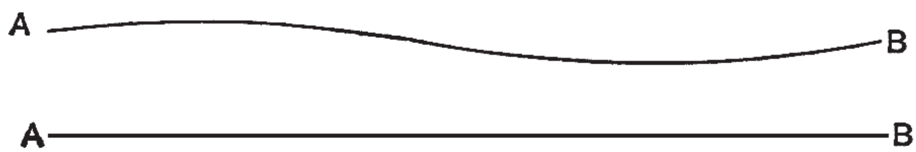
(ii) *The straight line which joins two points is the shortest distance between them; and there is only one such straight line.*

(iii) *Two straight lines cannot enclose a space.*

(iv) *If two straight lines cross one another they can only cut at one point.*

When you rule a straight line between two points **A** and **B**, you are said to **join AB**.

Test of straightness. We can find if a given line **AB** is straight or not by means of a copy of it made on tracing-paper. If by turning the tracing either *round* or *over* we can in any way make the given line and the tracing enclose a space, then the given line is not straight. But if in *all* such positions the tracing can be made to fit exactly over the given line throughout its whole length, then we may conclude that the latter is straight. Apply this test to the two lines drawn below.



Planes. Several different kinds of surfaces have been shown to you, and you have noticed that some are rounded or curved, and some are **plane,** that is to say, *flat.* How can we tell a plane surface from a curved one?

Lay the straight-edge of a ruler on a table, and notice that the *whole length* of the edge always rests upon the surface, *in whatever position the ruler is placed.* But if the ruler is placed in the hollow of a basin, only the ends rest on the surface: or again, if the straight-edge is laid against a sphere, it touches the surface at one point only.

Thus *a surface is* **plane** *when the straight line joining* **any** *two points on it lies entirely on the surface.*

Note. There are some curved surfaces, such as those of a *cylinder* and *cone,* along which a ruler will lie in *certain directions,* but not in *all* directions. The teacher should illustrate this with his models.

Exercise 1. What is the least number of *straight* lines that can enclose a space?

Rule *three* straight lines so as to enclose a space.

Rule *four* straight lines so as to enclose a space.

Exercise 2. Can two *curved* lines enclose a space? If so, make a drawing either free-hand or with compasses, showing a space enclosed by two curved lines.

Exercise 3. Can *one* curved line enclose a space? Make a drawing to illustrate your answer, either free-hand or with your compasses.

Exercise 4. Mark a point on your paper, and call it **A**. How many straight lines, having different directions, can be drawn through the point **A**?

Rule *five* straight lines passing through **A**.

Exercise 5. Mark two points **A** and **B**. *Join* **AB**. Observe that the position of a *straight* line is fixed if we know *two* points through which it passes. How many *curved* lines can be drawn from **A** to **B**? Draw *three* such lines, either free-hand or with your compasses.

Exercise 6. Mark *three* points **A**, **B**, and **C**, placing them so that they do not lie all in a straight line. How many straight lines can be drawn by joining these points in pairs? Draw all these lines.

Exercise 7. Repeat Exercise 6, but take *four* points **A**, **B**, **C**, and **D**, no three of which lie in a straight line, and join them in pairs.

CHAPTER II

MEASUREMENT OF STRAIGHT LINES

In practical geometry you will frequently have to measure the lengths of the lines you draw. For this purpose you have a scale which shows inches along one of its edges, each inch being divided into 10 equal parts: along another edge *centimetres* are marked, and each centimetre is also divided into 10 equal parts or *millimetres*.

Begin by carefully noticing the length of 1 inch and of 1 centimetre, so that you may be able to guess pretty nearly (even without measurement) how many inches or how many centimetres there are in a given line.

In writing down your measurements use the following abbreviations:

in. for *inch; cm.* for *centimetre; mm.* for *millimetre.*

Inches may also be denoted by the mark (″). Thus 3″ means 3 *inches.*

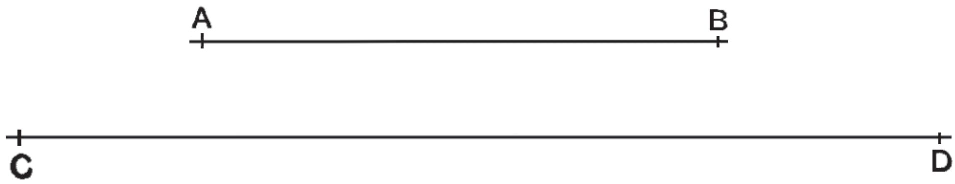
The units on your scale are divided into *tenths* in order that your measurements may be recorded *decimally:* Thus

(i) *Three and seven-tenths inches* should be written 3.7 in., or 3.7″.

(ii) *Eight-tenths of an inch* should be written 0.8 in., or 0.8″

(iii) *Five centimetres four millimetres* should be written 5.4 cm.

Exercise 1. Measure the lengths of **AB** and **CD** in inches and tenths of an inch.



Exercise 2. Measure the above lines **AB** and **CD** as nearly as you can in centimetres and millimetres.

Exercise 3. Measure **AX** and **XB** in inches and tenths of an inch, and add your results together. Test your work by measuring **AB**.

Record your results thus: By measurement, **AX** = in.
By measurement, **XB** = in.
By addition, **AX + XB** = in.
By measurement, **AB** = in.

Exercise 4. Measure **AX** and **XB** in centimetres and millimetres, and find their difference. Test your result by measuring **AB**.

Record your results as above.

Exercise 5. (i) Measure **AB**, **AX**, and **XY** in inches and tenths of an inch: hence reckon the length of **YB**, and test your result by measurement.

(ii) Measure **AY**, **YB**, and **XB** in centimetres, and hence find **AY + YB − XB**. What line should you now measure to test your result?

In each case arrange your results in tabular form.

Exercise 6. Draw straight lines to show the following lengths:

2.6 in.,	5.0 cm.,	1.8″,	4.7 cm.,	0.8 in.
8.2 cm.,	3.1″,	0.7 cm.,	9 mm.,	33 mm.

(Subdivision of a line by measurement)

Exercise 7. How would you find the middle point in the length of a strip of paper (i) by folding, (ii) by measurement?

Exercise 8. Draw a line **AB** of length 3″. What is the length of half **AB**? From **AB** mark off one-half, and thus find **O** the middle point of **AB**. Test your work by measuring **OB**.

A straight line is said to be **bisected** when it is divided into *two equal* parts.

Exercise 9. Draw a line **AB** of length 8.1 cm. What is the length of one-third of **AB**? With your dividers step off along **AB** one-third of its length, and thus divide **AB** into three equal parts.

A straight line is said to be **trisected** when it is divided into *three* equal parts.

Exercise 10. Draw a line **AB** of length 7.2 cm. By measurement, as explained above, cut off from it **AP** equal to *half* **AB**, and **AQ** equal to *one-third* **AB**. Find with your dividers how many times **PQ** is contained in **AB**. Explain your result by finding the value of ½ – ⅓.

(Comparison of 1 inch with 1 centimetre)

Exercise 11. Take 1 inch in your dividers, and apply them to your centimetre scale. How many centimetres and millimetres do you find in 1 inch?

It is impossible even with the greatest care to measure a length with perfect correctness; but the error is likely to be smaller *in proportion* in measuring a longer than in measuring a shorter length.

Exercise 12. Find the length of 1 inch in centimetres by measuring a length of 4 inches, and then dividing the result by 4.

Thus 4 inches = cm.
∴ 1 inch = cm.

Exercise 13. Measure a length of 1 centimetre against your inch scale. Then measure a length of 10 centimetres, and divide the result by 10. Compare the two equivalents of 1 cm., and observe that the second is likely to be the more correct.

(Distances represented by Lines drawn to Scale)

A map or plan is a small but exact flat copy of the country or ground it represents. Therefore by measuring on a map the distance between two dots which mark certain towns, we may reckon the real distance between the towns themselves, provided we know the *scale* on which the map is drawn. For instance, if 1 inch measured on the map stands for 10 miles, then 2″ stands for 20 miles; 4.5″ for 45 miles; and so on. Such a map is said to be drawn on *the scale of* 10 *miles to* 1 *inch.*

Exercise 14. The plan of an estate is drawn on the scale of 75 yards to 1 inch:

(i) What distance on the ground is represented by 3.6″ on the map?

Here 1 inch represents 75 yards;
∴ 3.6 inches . . . . 75 yards × 3.6
= 270 yards.

(ii) What length on the map will represent 405 yards?

Here 75 yards are represented by 1 inch;
∴ 405 yards . . . . 1 inch × $405/75$
= 5.4″.

Exercise 15. A plan is drawn on the scale of 100 metres to 1 centimetre:

(i) What actual distances are represented on the map by 4.0 cm., by 5.6 cm., by 0.8 cm.?

(ii) Draw lines to represent 450 metres, 720 metres, 580 metres, and 60 metres.

Exercise 16. On a map in which 1″ stands for 20 miles, the distance between Halifax and Hull is represented by 3.2″. What is the actual distance?

Bedford is 86 miles from Norwich: how far apart would they be on the map?

Exercise 17. The points marked **Sa**., **So.**, **W** represent the positions of Salisbury, Southampton, and Winchester on a map whose scale is 10 miles to 1 inch.

Find by measurement and reckoning the actual distances between Salisbury and Winchester, Winchester and Southampton, Southampton and Salisbury.

[In the following Exercises plans are to be drawn on squared paper ruled to tenths of an inch, and the results are to be got by measurement and reckoning.]

Exercise 18. I walk 4 miles due North, then 3 miles due East. Draw a plan to show my journey, making 1 in. stand for 1 mile; then by measurement find how far I am from my starting point.

Exercise 19. Draw the ground-plan of a room, 30 feet long by 20 feet wide, making 1″ represent 10 feet. Find as nearly as you can the actual distance between two opposite corners.

Exercise 20. An upright pole, standing 25 feet high, is stayed by a rope carried from the top to a point on the ground 15 feet from the foot of the pole. Represent this by a drawing (scale 10 feet to 1 inch); and find the length of the rope.

Exercise 21. A ladder reaches a window-sill 15 feet high, and the foot of the ladder rests on the ground 8 feet from the front of the house. Draw a plan (scale 5 feet to 1 inch), and use it to find the length of the ladder.

Exercise 22. Looking Eastward from my house, I see a church tower which I know to be 2 miles distant. Looking North I see a second tower 1½ miles away. Draw a plan (scale 1 mile to 1 inch), and find how far the towers are apart.

Exercise 23. A ship on leaving harbour sails 22 miles South, then again 22 miles West. Represent her course on the scale of 10 miles to 1 inch, and find her distance from the harbour.

Exercise 24. In rowing across a river 48 metres wide, a man was carried 16 metres down stream. Represent this on a plan (scale 20 metres to 1 inch); hence find the distance between the starting point and landing-point.

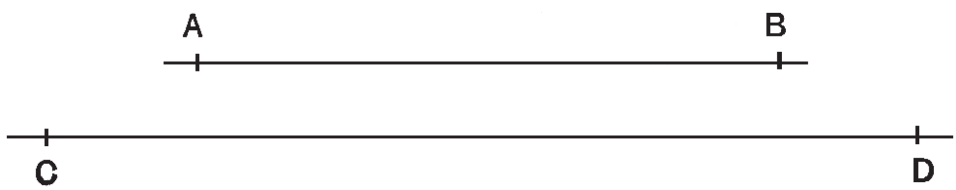
CHAPTER III

STRAIGHT LINES CONTINUED

⁂ This Section may be postponed for revision.

If you measure the same line in several ways, some of your results may be a little too large and some a little too small. The *average* of your results is likely to be nearer the truth than any single result. To find the average, add your results together, and divide their sum by the number of them.

Exercise 1. Measure **AB** in inches and also in centimetres; and hence express 1 inch in terms of cm. and mm.

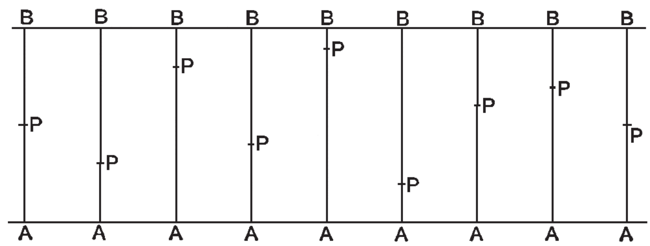


Measure **CD**, and repeat the process. Now find the average of your two results.

(Judging Lengths, Errors, Relative Errors)

It is important that you should train your eye to subdivide any unit of length into *tenths* without actual measurement. Remember that *one-half* = *five*-tenths: this gives a standard to judge by. Fix your eye on the middle point, and mentally divide each half into five equal parts.

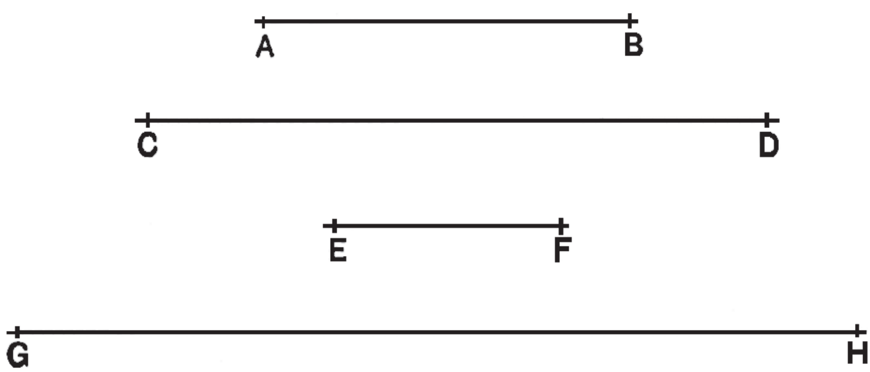
Exercise 2. The lines marked **AB** are all 1 inch long. State in each case how many tenths of an inch there are in **AP**; then verify your answer by measurement.



Exercise 3. Draw six lines each 1 inch long, calling one end **A**. Then mark a point **P** in each (without measurement) so that, as nearly as you can judge, **AP** may be in succession 0.4″, 0.7″, 0.2″, 0.9″, 0.3″, 0.6″.

Check your attempts by measurement.

Exercise 4. (i) Judge as nearly as you can in inches and centimetres the lengths of the lines given below.



Check your estimates by measurement, and tabulate the results as on the next page, leaving the last column blank for the present.

	Measured length.	Estimated length.	Actual error.	Percentage error.
AB {	in. cm.	in. cm.	in. cm.	

⁂ *Other lines of greater length and not all horizontal should be given by the teacher.*

(ii) Draw lines as nearly as you can judge without measuring to show 6 cm., 2.0″, 8 cm., 3.5″. Measure your attempts; note your errors, and tabulate the results.

In judging the importance of an error we do not care so much whether it is large or small, as whether it amounts to a large or small fraction of the quantity we are estimating. For instance: suppose that in guessing the length of a line whose real length is 5 cm. we are wrong by 1 cm.; while in guessing a line 20 cm. long we are wrong by 2 cm. The actual error in the latter case is greater than in the former, but it is really of less importance. For in the second case the error is only *one-tenth* of the real length, that is, *one in ten*; while in the first case it is *one-fifth,* or *one in five.* Errors thus measured as fractions of the true value are called **relative errors**: and it is convenient to reduce them to a fixed standard, as so many *in one-hundred,* or so many *per cent.* Take the following case:

Real length.	Estimated length.	Actual error.	Percentage error.
8.0 cm.	7.5 cm.	0.5 cm.	

Here on a real length of 8 cm. the error is 0.5 cm.

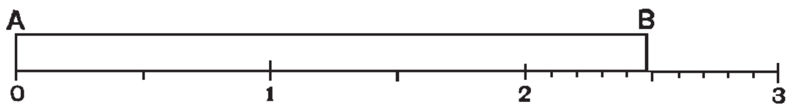
$\therefore$ . . . . 100 cm. the error is 0.5 cm. $\times \frac{100}{8}$

$= 6\frac{1}{4}$ cm.

That is, the error is at the rate of *6¼ in one hundred,* or *6¼ per cent.* We may now enter 6¼ in the last column.

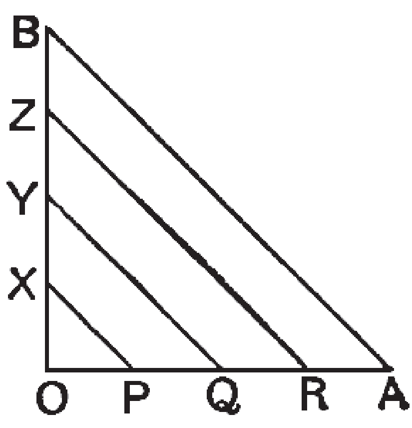
Exercise 5. Fill up the percentage column in Exercise 4, giving the percentage correct to one decimal figure.

Hitherto the lines which you have had to measure in inches and tenths of an inch have contained an exact number of tenths. This will not always be so. For example



the line **AB** above (where ruler is enlarged) represents a line that is more than 2.4″ and less than 2.5″. In this case we may mentally divide the tenth in which **B** falls into *ten* equal parts, that is to say, into *hundredths of an inch,* and judge as nearly as we can how many of these hundredths are to be added to 2.4. In this instance about *seven*-hundredths should be added, so that the length of **AB** is nearly 2.47″.

Exercise 6. Draw on squared paper a figure like that below, making **OA** and **OB** each 2″ long. Put **P**, **Q**, **R** and **X**, **Y**, **Z** at the half-inch divisions; then measure **AB**, **RZ**, **QY**, **PX** as nearly as you can in *inches, tenths* and *hundredths.*



CHAPTER IV

CIRCLES

Mark a point **O** on your paper. Take a distance of 5 cm. between the points of your compasses; then, placing the steel point at **O**, turn the compasses between your fore-finger and thumb so as to draw a curved line with the pencil-point.

As the curved line is being traced out, notice carefully that the pencil-point always keeps the same distance from **O**. What distance? Notice also that the pencil returns to its starting point, so as to close the curve. Why is this?

The curve you have thus drawn is called a **circle,** and the point **O** is its **centre.** Sometimes the word *circle* means the space enclosed by the curve, and then the curve itself is said to be the **circumference** of the circle.

Exercise 1. Mark a few points, say four, anywhere on the circumference of the circle you have drawn: call them **A**, **B**, **C**, **D**. Join **OA**, **OB**, **OC**, **OD**. How do you know that these lines are all equal! Tell their length without measuring them.

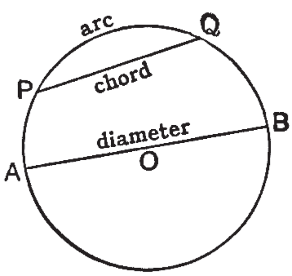
Straight lines drawn from the centre of a circle to its circumference are called **radii.** All the radii of a circle are equal.

Exercise 2. Mark a fixed point **O** on your paper: then with your compasses mark any *four* points whose distance from **O** is 2.0″. How many points could you mark whose distance from **O** is 2.0″? Draw a curve to pass through all of them.

Exercise 3. Suppose a point **X** is taken 1.7″ from the centre of the circle you have just drawn (Exercise 2); another point **Y** is 2.0″, and a third point **Z** is 2.3″ from the centre. Which of these points is on the circumference? Which outside it? Which within it?

Exercise 4. Invent some other means, besides compasses, by which a circle could be drawn having a fixed point **O** as centre.

Exercise 5. Now explain in your own words what a circle is, telling how the circumference is related to the centre.



Taking a point **O** as centre, draw a circle with a radius of 1.5″. Then through the centre **O** draw any straight line ended each way by the circumference. Such a line is called a **diameter,** and is represented in the Figure by **AB**.

Exercise 6. What is the length of **AB** in your drawing? Answer this without measuring. Are all diameters of a circle equal?

Now carefully cut your circle out, and fold it about the diameter **AB**, thus dividing the circle into two parts. Do you find that one part fits exactly over the other? If so, this shows that the two parts *are of the same size and shape*. Flatten out the circle; rule any other diameter, and fold the circle about it as before. Again you find that one part fits exactly over the other. All this we express by saying that a circle is **symmetrical** about any diameter.

The two equal parts into which a circle is divided by a diameter are called **semi-circles.**

An **arc** (i.e. *bow*) is any part of the circumference of a circle,

A **chord** (i.e. *string*) is the straight line joining the ends of an arc.

Exercise 7. Draw a circle of diameter 3.0″, and on the circumference mark a point **X**. From **X** draw two chords, one 1.5″ long, the other 2.0″ long. What is the length of the longest chord in this circle?

Exercise 8. In the Figure on the opposing page notice that the chord **PQ** divides the circumference into *two* arcs. Point them out. Can a chord ever cut off two *equal* arcs? Which is the longer line, an arc, or the chord which joins its ends?

(Two or more circles, Intersection of circles)

Exercise 9. Mark a point **O** on your paper, and from **O** as centre draw three circles, one of radius 3.5 cm., the next of radius 4.0 cm., the third of radius 4.5 cm. Notice that the circumferences do not cross or cut one another. Why not?

Circles which have the same centre are said to be **concentric.**

Exercise 10. (i) Take two points **A** and **B**, 7 cm. apart. With **A** as centre draw a circle of radius 4 cm.; and with **B** as centre draw a circle of radius 2 cm. Explain why each circle is outside the other. What is the shortest distance between the circumferences?

(ii) Again take two points **A** and **B**, 7 cm. apart; and, as before, with **A** as centre draw a circle of radius 4 cm. But this time draw from centre **B** a circle of radius 5 cm. Why do these circles overlap? At how many points do the circumferences cut one another?

(iii) Once more take two points **A** and **B**, 7 cm. apart, and with **A** and **B** as centres draw two circles, one of radius 4 cm., the other of radius 3 cm. Do the circumferences cross one another? Do they meet? If your work is carefully done, the two circles just *touch* one another. Where is the touching point? Say why.

Exercise 11. Take two points **A** and **B**, 2 cm. apart; and with centre **A** draw a circle of radius 5 cm. With centre **B** draw a circle of radius 3 cm. How does this circle meet the first, and where is the meeting-point?

Exercise 12. Can you draw two circles which cut one another at more than two points? Try.

Exercise 13. Take two points 3″ apart, and call them **A** and **B**. With centre **A** and radius 2½″ draw a circle. With centre **B** and radius 2″ draw a second circle. Call the points at which the circles cut one another **P** and **Q**. How far is **P** from **A** and from **B**? How far is **Q** from **A** and from **B**?

Exercise 14. Take two points **A** and **B**, 8 cm. apart. Find with your compasses a point which is 6 cm. from **A** and also 6 cm. from **B**. Can you find more than one such point? How many?

Exercise 15. Draw a line 2.5″ long, and find with your compasses a point that is 2.0″ from each end. How many such points are there?

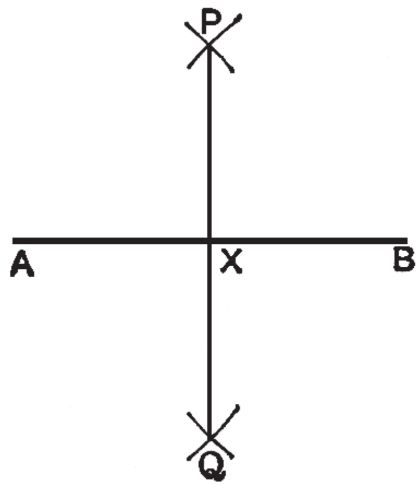
Exercise 16. Take two points **X** and **Y**, 9 cm. apart. Find a point which is 6 cm. from **X** and 5 cm. from **Y**. How many such points are there?

Exercise 17. Draw a line 3.3″ long, and find two points each of which is 2.2″ from one end and 1.8″ from the other.

Exercise 18. Two forts defending the mouth of a river, one on each side, are 10 kilometres apart: their guns have an effective range of 6000 metres. Draw a plan (scale 1 km. to 1 cm.) showing what part of the river is exposed to fire from both forts.

PROBLEM 1

To bisect a straight line **AB** *with ruler and compasses.*



[The given straight line **AB** may be of any length: about 3″ to 4″ will be convenient, but do not measure it.]

Construction. Take in your compasses any length that appears to you to be greater than half **AB** (say about 2½″); and then with centre **A** draw arcs on each side of **AB**.

Again with centre **B**, and with the *same radius* as before, draw arcs to cut the first arcs as shown in the Figure. Call the cutting points **P** and **Q**.

Join **PQ**, and put **X** at the point where this line crosses **AB**.

Now take **AX** in your dividers, and see if **BX** is equal to it.

(Further Tests)

(i) Mark the points **A**, **B**, and **X** on tracing-paper, and turning it round, place the trace of **A** on **B**, and the trace of **B** on **A**. Where does the trace of **X** fall? How does this experiment show that **AB** has been bisected at **X**?

(ii) If the arcs drawn from centre **B** had a greater radius than those drawn from centre **A**, would **X** still be the middle point of **AB**? If not, towards which end of **AB** would **X** lie? Take your compasses and try. You see then that **X** is the middle point of **AB** *because we have worked from centre* **B** *in exactly the same way as from centre* **A**.

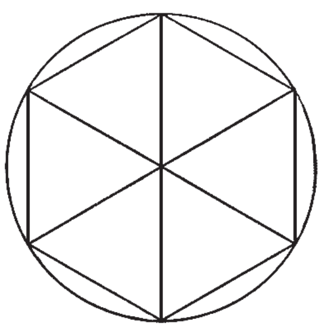
(iii) Why did we take a radius *greater* than half **AB**? What would have happened if the radius had been *less* than half **AB**? or exactly half **AB**? Take your compasses and try.

Exercise 19. Draw a line 8.5 cm. long, and bisect it with ruler and compasses. Test your drawing with the dividers.

Exercise 20. Draw a line 3.4″ long. Find the middle point **X** *by measurement.* Now bisect **AB** *by construction,* and see if the line **PQ** passes through **X**.

Exercise 21. Draw a line 9.6 cm. long. Bisect it by construction; then bisect each half.

Draw a circle, say of radius 2.0″, and with *the same radius* mark off points round the circumference. How many steps can you thus take? **Six** exactly. Are the *arcs* which you thus cut off each 2″ in length? Are they more or less than 2″? Join the points of division in order. Are the *chords* each 2″ in length? Why so? Join the centre to each point of division, and thus complete the pattern shown in the margin.



Exercise 22. Invent some simple experiment, for instance by cutting out, or folding, or by means of a tracing, to show that the six arcs are of equal length (though not 2″). Try to find the length of one of these arcs by laying a thread along it, straightening the thread out before measurement.

Exercise 23. Draw the patterns of which small copies are given below. Your drawings should be twice the size of the copies.

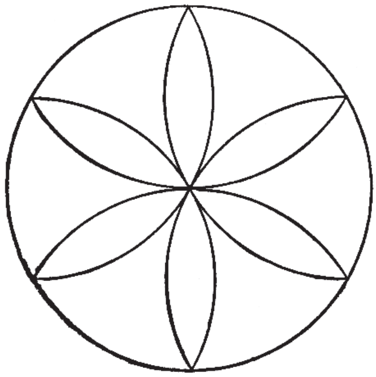


Fig. 1.

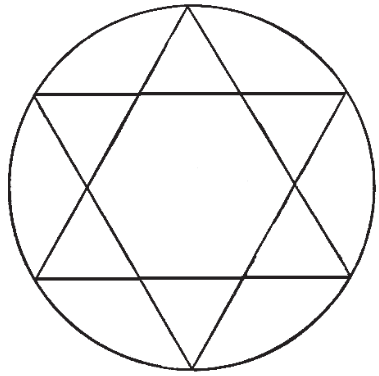


Fig. 2.